Math 208 Midterm I

October 20, 2023

NAME:

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STUDENT ID NUMBER:

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UW EMAIL:

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2	/20
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Total	/100

Instructions.
[1]

- For each problem below give a carefully explained solution using the vocabulary and notation from class. A correct answer with no supporting work or explanation will receive a zero.

- If a solution involves a numerical answer, collect all terms and reduce all fractions. Put a box around your final answers.

- You are allowed a simple calculator and notesheet. Other notes, electronic devices, etc are not allowed. Take a few pencils from your pencil case out and put all other items away for the duration of the exam.

- All the questions can be solved using (at most) simple arithmetic. (If you find yourself doing complicated calculations, there might be an easier solution...)

- Raise your hand if you have any questions or spot a possible error.

Good luck!

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[1]Test code: 8745

(1) Consider following system of equations to answer the questions below.

$$\begin{aligned} x_1 + 6x_2 + 4x_3 + x_4 &= 12 \\ x_2 + x_3 + x_4 &= 1 \\ x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_4 &= 2 \end{aligned}$$

(a) (5 points) Write down the augmented matrix for this system.

(b) (15 points) Solve this system by computing the REDUCED row echelon form for the augmented matrix by Gauss-Jordan elimination and label each step. (Hint: think about what steps to do first to make this process more efficient.)

(2) The Math Minded Knitters Consortium (MMKC) are making hats this winter. They use three types of yarn with different compositions of materials determined by the table below. Each type of yarn is the same weight.

Materials	Brand A	Brand B	Brand C
merino wool	70%	80%	90%
possum	10%	10%	10%
silk	10%	0%	0%
recycled plastic	10%	10%	0%

(a) (5 points) Write down the matrix equation that would determine the total number of grams of wool, possum, silk, and recycled plastic that are in a grams of Brand A yarn, b grams of Brand B yarn, and c grams of Brand C yarn. Explain the notation in the right side in your matrix equation for the members of MMKC.

(b) (5 points) Use your matrix equation to determine how many grams of each material are in a hat with 50grams of Brand A yarn, 20 grams of Brand B yarn, and 30 grams of Brand C yarn?

(c) (10 points) Can MMKC make 100 gram hats with 83 grams of wool, 10 grams of possum, 3 grams of silk, and 4 grams of recycled plastic? If so, how much of each brand of yarn should they use? If not, explain why not.

(3) Consider the matrices A and B in each question below.

$$A = \begin{bmatrix} 1 & -1 & 2 & 8 & 1 \\ 0 & 1 & -3 & 8 & 0 \\ 0 & 0 & 1 & 8 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 & 2 & 1 & 1 \\ 0 & 7 & -3 & 10 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) (5 points) Which subsets of columns of A are linearly dependent? Explain your answer using an Echelon Test Theorem.

(b) (5 points) Which subsets of columns of B span all of $\mathbb{R}^4$? Explain your answer using an Echelon Test Theorem.

(c) (5 points) Compute $C = A + 4B$.

(d) (5 points) Which subsets of columns of C are linearly dependent and which ones span $\mathbb{R}^4$? Explain your answer.

(4) Let S be the subspace of $\mathbb{R}^3$ determined by intersecting the plane given by $x - y + z = 0$ and the plane given by $x + y = 0$.

(a) (5 points) What familiar type of geometrical object is S?

(b) (5 points) What column vectors are in S?

(c) (10 points) Give the equation of a plane that does not intersect S. Explain why the intersection is empty.

(5) Consider the vector equation

$$w \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix}$$

(a) (10 points) What is the general solution?

(b) (10 points) Give a specific solution that involves a nonzero combination of all 4 vectors.

Bonus: (2pt) What have you found easiest and hardest so far in Math 208?

Do you wish the pace to go FASTER, SLOWER, or ABOUT THE SAME?

(Or if you don't want to answer those two questions, draw a picture involving vectors here.)